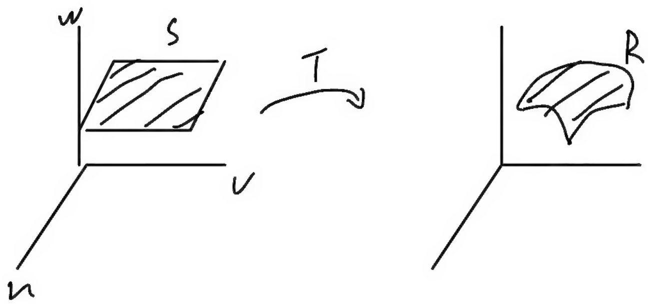


$T(u,v,w) = (x(u,v,w),\ y(u,v,w),\ w(u,v,w))$

Jacobian of T:

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{bmatrix}$$

$$\iiint_R f(x,y,z)\, dV_R$$

$$\Downarrow$$

$$\iiint_S f(x(u,v,w),\ y(u,v,w),\ z(u,v,w)) \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| dV_S$$

Ex. Derive the formula for the triple integral of sphereical

$x = \rho \sin\varphi \cos\theta$

$y = \rho \sin\varphi \sin\theta$

$z = \rho \cos\varphi$

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \sin\varphi\cos\theta & -\rho\sin\varphi\sin\theta & \rho\cos\varphi\cos\theta \\ \sin\varphi\sin\theta & \rho\sin\varphi\cos\theta & \rho\cos\varphi\sin\theta \end{vmatrix}$$

$$\begin{bmatrix} \cos\varphi & 0 & -\rho\sin\varphi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\varphi & 0 & -\rho\sin\varphi \\ \sin\varphi\cos\theta & -\rho\sin\varphi\sin\theta & \rho\cos\varphi\cos\theta \\ \sin\varphi\sin\theta & \rho\sin\varphi\cos\theta & \rho\cos\varphi\sin\theta \end{bmatrix}$$

$$= \cos\varphi \cdot (-\rho\sin\varphi\sin\theta \cdot \rho\cos\varphi\sin\theta) - |\rho\cos\varphi\cos\theta \cdot \rho\sin\varphi\cos\theta|$$

$$-\rho\sin\varphi\,(\rho\sin^2\varphi\cos^2\theta + \rho\sin^2\varphi\sin^2\theta)$$

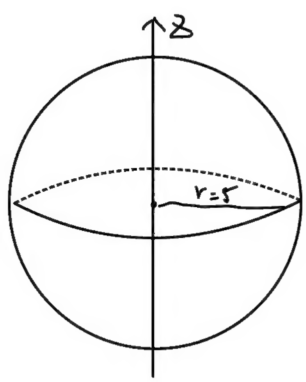
$$= -\rho^2\cos^2\varphi\sin\varphi - \rho^2\sin^3\varphi$$

$$= -\rho^2\sin\varphi$$

$$\boxed{\begin{array}{c} \rho \geq 0 \\ 0 \leq \theta \leq 2\pi \\ 0 \leq \varphi \leq \pi \end{array}}$$

$$|-\rho^2\sin\varphi| = \rho^2\sin\varphi \qquad 0 \leq \varphi \leq \pi$$

(V) compute $\iiint_R (x^2+y^2+z^2)\,dV$, where R is a solid ball of radius of 5 centered at origin



$\rho \geq 0$

$0 \leq \theta \leq 2\pi$

$0 \leq \varphi \leq \pi$

$x = \rho\sin\varphi\cos\theta$

$y = \rho\sin\varphi\sin\theta$

$z = \rho\cos\varphi$

$x^2+y^2+z^2=\rho^2$

$\iiint_R (x^2+y^2+z^2)\, dV = \int_0^{\pi}\int_0^{2\pi}\int_0^5 (\rho^2)^2 \cdot \rho^2 \sin\varphi \, d\rho\, d\theta\, d\varphi$

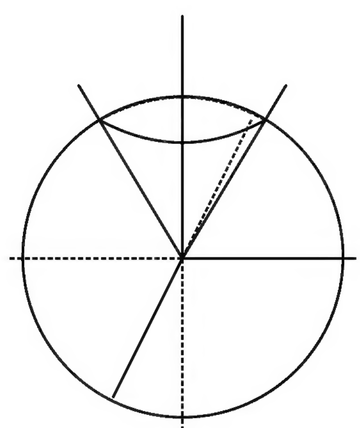
$= \left(\int_0^5 \rho^6 \, d\rho\right)\left(\int_0^{2\pi} d\theta\right)\left(\int_0^{\pi} \sin\varphi \, d\varphi\right)$

$= \left[\frac{1}{7}\rho^7\right]_0^5 \left[\theta\right]_0^{2\pi} \cdot \left[-\cos\varphi\right]_0^{\pi}$

$= \frac{5^7}{7} \cdot 2\pi \cdot 2$

$= \frac{4\pi}{7} 5^7$

compute $\iiint_R y^2 z \, dV$ where R is the region above the cone with the cone point at the region and making an angle of $\frac{\pi}{3}$ radians with positive z-axis, and inside the sphere of radius 2 (centered at origin)



$0 \le \rho \le 2$

$0 \le \theta \le 2\pi$

$0 \le \varphi \le \frac{\pi}{3}$

$$\iiint_R y^2 z\, dA = \int_0^{\frac{\pi}{3}} \int_0^{2\pi} \int_0^2 \rho^2 \sin^2\varphi \sin^2\theta \cdot \rho\cos\varphi \cdot \rho^2 \sin\varphi \, d\rho\, d\theta\, d\varphi$$

$$= \int_0^2 \rho^5 d\rho \cdot \int_0^{\frac{\pi}{3}} \sin^3\varphi \cdot \cos\varphi \, d\varphi \cdot \int_0^{2\pi} \sin^2\theta \, d\theta$$

$$= \frac{32}{3} \cdot \int_0^{2\pi} \frac{1-\cos 2\theta}{2} d\theta \cdot \int_0^{\frac{\sqrt{3}}{2}} u^3 du$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\pi \qquad\qquad \frac{1}{4} \cdot \frac{9}{16}$$

$$= \frac{3}{2}\pi$$

compute $\iiint_R xy^2 z$ where R is the region bounded by surfaces $x = 4y^2 + 4z^2$ and plane $x = 4$

$4y^2 + 4z^2 \le x \le 4$ $\qquad$ $y = r\cos\theta$

$y^2 + z^2 = 1$ $\qquad$ $z = r\sin\theta$

$-1 \le y \le 1$ $\qquad$ $0 \le r \le 1$

$-1 \le z \le 1$ $\qquad$ $x = x$

$$\iiint_R x y^2 z \, dA$$

$$= \int_0^{2\pi} \int_0^1 \int_{4r^2}^4 x\, r^3 \cos^2\theta \sin\theta \cdot r \, dx\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^1 8r^4 \cos^2\theta \sin\theta - 8r^8 \cos^2\theta \sin\theta \, dr \, d\theta$$

$$= \int_0^{2\pi} \frac{8}{5} \cos^2\theta \sin\theta - \frac{8}{9} \cos^2\theta \sin\theta \, d\theta$$

$$= 0$$